Relativistic dynamics and proper force - proper power

According to the Newton's first and second laws of Newton the basic concept and the mathmatical form of force in terms of momentum are obtained respectively.

Classically the second law gives

$$F = \frac{dp}{dt} \longrightarrow (1)$$

We will see that the above classical relation retains or holds it validity in relativistic mechanics also.

For a small displacement dl the work done, due to the application of the force (1) is

$$W = \int F \cdot dl \longrightarrow (2)$$

$$\Rightarrow W = \int \frac{dp}{dt} \cdot dl = \int \frac{dp}{dt} \cdot \frac{dl}{dt} dt = \int \frac{dp}{dt} \cdot u\, dt \longrightarrow (3)$$

Now $\frac{dp}{dt} \cdot u = \frac{d}{dt}(mu) \cdot u = \frac{d}{dt}\left(\frac{m_0 u}{\sqrt{1 - u^2/c^2}}\right) \cdot u$

$$\therefore \frac{dp}{dt} \cdot u = \left\{ \frac{m_0 u}{\left(\sqrt{1 - \frac{u^2}{c^2}}\right)^3} \cdot \frac{du}{dt} \right\}$$

[Differentiating the bracketed term as $\frac{d}{dt}\left(\frac{u}{v}\right)$ form]

$$\longrightarrow (4)$$

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The R.H.S of the above equation can be replace as

$$\frac{dp}{dt}\cdot v = \frac{d}{dt}\left(\frac{m_0c^2}{\sqrt{1-\frac{u^2}{c^2}}}\right) = \frac{d(mc^2)}{dt}$$

As $\frac{m_0c^2}{\sqrt{1-\frac{u^2}{c^2}}} = mc^2$

$\therefore \frac{dp}{dt}\cdot v = \frac{dE}{dt}$ ⟶ (5)

Rough

$$\frac{d}{dt}\left(\frac{m_0c^2}{\sqrt{1-\frac{u^2}{c^2}}}\right)$$

$$= -\frac{1}{2}\frac{(m_0c^2)\left(-\frac{2u}{c^2}\right)}{\left(1-\frac{u^2}{c^2}\right)^{3/2}}\frac{du}{dt}$$

$$= \frac{m_0u}{\left(1-\frac{u^2}{c^2}\right)^{3/2}}\frac{du}{dt}$$

= RHS of eqn (4)

Using (5) in equation (3) we have

$$W = \int \frac{dE}{dt}\cdot dt = E_{final} - E_{initial} \longrightarrow (7)$$

Here E may be the total energy or kinetic energy as the rest energy m_0c^2 is a const and remains invariant under any relativistic change.

∴ The classical work energy theorem holds relativistically (eqn (7)), ie the net work done on a particle is equal to the increase in kinetic energy of the particle.

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From equ^{n} (1) if the 3 component of the force are considered,

$$F_x' = \frac{dp_x'}{dt'} \quad \text{[In primed system]}$$

$$= \frac{(dp_x - \frac{v}{c}dp^0)/\sqrt{1-v^2/c^2}}{(dt - v\,dx/c^2)/\sqrt{1-v^2/c^2}}$$

$$= \frac{\gamma(dp_x - \beta dp^0)}{\gamma(dt - \frac{\beta}{c}dx)}$$

$$= \frac{\frac{dp_x}{dt} - \beta\frac{dp^0}{dt}}{1 - \frac{\beta}{c}\frac{dx}{dt}}$$

$$= \frac{F_x - \frac{\beta}{c}\frac{df}{dt}}{1 - \frac{\beta u_x}{c}} \longrightarrow (8a)$$

In 4D Standard form

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\beta = \frac{v}{c}$$

p^0 = time of 4th component of momentum

$$\frac{dp_x}{dt} = F_x$$

$$F_y' = \frac{dp_y'}{dt'} = \frac{dp_y^0}{\gamma(dt - \frac{\beta}{c}dx)} = \frac{dp_y^0/dt}{\gamma(1 - \frac{\beta u_x}{c})} = \frac{F_y}{\gamma(1 - \frac{\beta u_x}{c})} \longrightarrow (8b)$$

By $F_z' = \frac{dp_z'}{dt} = \frac{F_z}{\gamma(1 - \frac{\beta u_x}{c})} \longrightarrow (8c)$

All the 3 equation (8a), (8b), (8c) may be expressed in a single equation by introducing the new proper force (K^μ). It would be the derivative of momentum with respect to proper time.

$$\boxed{K^\mu \equiv \frac{dp^\mu}{d\tau}} \longrightarrow (9)$$